Design of periodic event-triggered control for nonlinear systems using overapproximation techniques

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1 Introduction

We consider a dynamical system of the form

$$\dot{x} = f(x, u),$$

where $x \in \mathbb{R}^{n_x}$ is the state and $u \in \mathbb{R}^{n_u}$ the control input, with $n_x, n_u \in \mathbb{N}_{>0}$. We assume that a control law $u = k(x)$ is available, such that the closed loop in continuous time satisfies desired stability and performance specifications. However, on a digital platform, the control law cannot be implemented in continuous time. Instead, the input $u$ is only updated at discrete times $t_k$, $k \in \mathbb{N}$, and held constant between updates, leading to

$$\dot{x}(t) = f(x(t), k(x(t_k))) = f(x(t), k(x(t) + e(t))), \quad (2)$$

for all $t \in [t_k, t_{k+1})$, where $e(t) := x(t_k) - x(t)$ is reset to zero at each sampling instant. We can write (2) as

$$\dot{z}(t) = g(z(t)), \quad \text{for all } t \in [t_k, t_{k+1}) \quad (3)$$

$$z(t_{k+1}) = b(z(t_k)). \quad (4)$$

for all $k \in \mathbb{N}$, where $z$ consists of $x$ and $e$, and $g$ and $b$ are obtained by direct calculations.

In conventional systems, the jump times $t_k$, $k \in \mathbb{N}$, are determined a priori, purely based on time. In event-triggered control (ETC), the jump times are defined as

$$t_{k+1} = \min\{t > t_k \mid \Gamma(z(t)) \geq 0\}. \quad (5)$$

where $\Gamma$ is chosen such that ensuring $\Gamma(z(t)) \leq 0$ guarantees the required performance for the closed loop. The updates based on (5) are typically less frequent than those based on conventional time-triggered control, which saves valuable computation and communication resources.

2 From ETC to PETC

In (5), the function $\Gamma(z(t))$ has to be monitored continuously, which is also hard to accomplish on digital platforms. Therefore, a periodic event-triggered controller (PETC) [1] is of interest that will only carry out periodic time-triggered checks of a different event-triggering condition of the form $\hat{\Gamma}(z(t))$ but still ensures that $\Gamma(z(t)) \leq 0$ for all times $t \in \mathbb{R}_{\geq0}$, where $\hat{\Gamma}$ has to be designed. To be precise, the jump times in the envisioned PETC implementation are given by

$$t_{k+1} = \min\{t = t_k + nh \mid \hat{\Gamma}(z(t)) \geq 0, n \in \mathbb{N}_{>0}\}, \quad (6)$$

where $h > 0$ is the sampling period. The problem of designing $h$ and $\hat{\Gamma}$ that achieve this goal has been addressed in [1], where a design methodology is presented to approximately preserve the non-positivity of $\Gamma$ along the system’s solution. The objective of this work is to provide design guidelines for $\hat{\Gamma}$ and $h$ to exactly preserve $\Gamma(z(t)) \leq 0$ along the solutions to the corresponding PETC system. The proposed construction exploits ideas from [1], which show that under certain assumptions on differentiability of $g$ and $\Gamma$, the evolution of $\Gamma(z(t))$ can be upper bounded by a linear system. Using these results from [1] we can construct $\hat{\Gamma}$ by exploiting ideas from convex overapproximation [2], resulting in a systematic design procedure for PETC strategies for nonlinear systems.

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References
