Optimal Control of Uncertain Switched Systems Based on Model Reference Adaptive Control

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1 Introduction

Switched systems are an important subclass of hybrid systems that consists of subsystems with continuous dynamics and a rule to regulate the switching behavior between them. Switched systems appear in a wide range of applications, such as intelligent transportation systems and smart energy systems. Due to the desire to drive switched systems toward optimal behavior, e.g. maximization of traffic flow in traffic networks, minimization of energy consumption in energy systems, optimal control of switched systems has attracted a lot of attention. Solution of the Hamilton-Jacobi-Bellman (HJB) equation via Dynamic Programming has been proposed to address optimal control for switched systems. This, however, gives rise to the ‘curse of the dimensionality’. Recently, adaptive dynamic programming (ADP) has been adopted to solve optimal control problems for switched systems. ADP algorithms are capable of avoiding the ‘curse of the dimensionality’ and approximating the optimal control law via recursive relationships and soft computation technologies, like neural networks. Lu and Ferrari [1] train neural networks to approximate the optimal value function in a forward fashion. The weights are updated online by recursive relationships involving the objective function and its gradient with respect to the switching modes. Heydari and Balakrishnan [2] adopt ADP to obtain the optimal switching sequence and switching instants. A batch training algorithm is applied to update weights of the neural networks in a backward fashion.

2 Problem formulation and methodology

Approaches developed in literature for optimal control of switched systems require to some extent the knowledge of the system dynamics. For example, the knowledge of the system dynamics in a neighborhood set of the continuous system state trajectory is a prerequisite condition of the approach of [1]. The algorithm of [2] is based on known dynamics of the switched system and can be only implemented offline. In light of this, we propose to adopt a model reference adaptive control (MRAC) scheme to solve optimal control problems of uncertain LTI switched systems. MRAC is an adaptive scheme to obtain a control law that drives the output of an unknown system to track the response of a reference model. The unknown dynamics of switched systems can be expressed as

\[ \dot{x}(t) = A_i(t)x(t) + B_i(t)u(t) + d(t) \]
\[ y(t) = C_i(t)x(t) \]

where \( x, u, y \) and \( i \) are the state vector, the continuous control law, the output and the switching sequence respectively; \( d \) is a bounded disturbance. For every mode \( i \), the matrices \( A_i, B_i \) and \( C_i \) are unknown. A family of reference models, one for each mode, is also defined: the reference models have input \( r \), state \( \hat{x} \) and output \( \hat{y} \). Moreover, the retrospective cost function considering the past output difference is needed. Instead of solving the HJB equation, the inputs and objective function of switched systems are optimized based on asymptotically diminishing the difference online between the expected output of the reference models and the closed-loop output. Additionally, a family of Lyapunov functions, one for each subsystem, is considered to determine the corresponding switching actions. The stability of the switched systems will be guaranteed by optimizing the switching sequence so that the values of the Lyapunov function at switch-in points of one subsystem are smaller than those of previous switch-in points.

3 Conclusions and future work

To date, the optimal control problem of uncertain switched systems has not been solved satisfactorily. Every method proposed in literature, to the best of the authors’ knowledge, requires to some extent the knowledge of the system dynamics. In this work, we propose a new optimal control scheme for unknown switched systems based on model reference adaptive control.

References